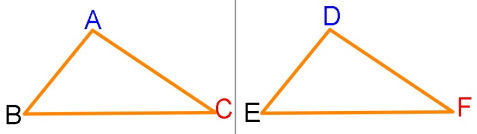
Section 4.1
Congruent Figures

Congruent figures

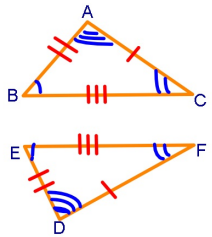
- figures that have the same size and same shape



Triangles ABC and DEF are congruent

Congruent Triangles

- two congruent triangles are congruent iff their vertices can be matched up so that corresponding parts (angles and sides) of triangles are congruent.



D ⟷ A
E ⟷ B
F ⟷ C

The corresponding parts are congruent

When referring to congruent triangles, we name their corresponding vertices in the same order. For the triangles above,

Δ DEF is congruent to ΔABC

Suppose we are given that ΔXYZ is congruent to ΔABC. From the definition of congruent triangles we know that

XY ≅ AB

∠X ≅ ∠A

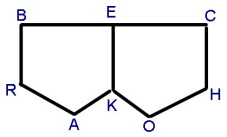
When the definition of congruent triangles is used to justify either of these statements, we use

corresponding parts of congruent triangles are congruent

Corr. parts of ≅ Δ's are ≅

CPCTC

Two polygons are congruent iff their vertices can be matched up so that their corr. parts are congruent.



BRAKE ≅ CHOKE

$\overline{EK}$ is called a common side

Suppose you know that $\triangle FIN \cong \triangle WEB$.

1. Name the three pairs of corresponding sides.
2. Name the three pairs of corresponding angles.
3. Is it correct to say $\triangle NIF \cong \triangle BEW$?
4. Is it correct to say $\triangle INF \cong \triangle EWB$?

The two triangles shown are congruent. Complete.

5. $\triangle ABO \cong$ ___?___

6. $\angle A \cong$ ___?___

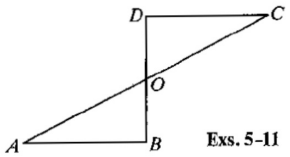
7. $\overline{AO} \cong$ ___?___

8. $BO =$ ___?___

9. Can you deduce that O is the midpoint of any segment? Explain.

10. Explain how you can deduce that $\overline{DC} \parallel \overline{AB}$.

11. Suppose you know that $\overline{DB} \perp \overline{DC}$. Explain how you can deduce that $\overline{DB} \perp \overline{BA}$.



Exs. 5–11

The pentagons shown are congruent. Complete.

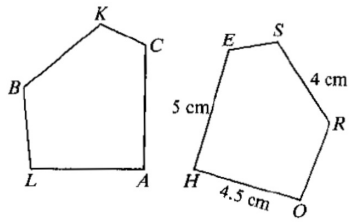
12. B corresponds to ___?___.

13. $BLACK \cong$ ___?___

14. ___?___ $= m\angle E$

15. $KB =$ ___?___ cm

16. If $\overline{CA} \perp \overline{LA}$, name two right angles in the figures.

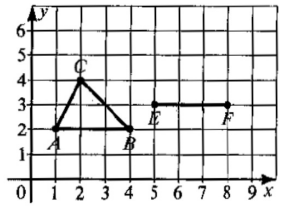


18. **a.** Name the coordinates of points A, B, and C.

b. Name the coordinates of a point D such that $\triangle ABC \cong \triangle ABD$.

19. Name the coordinates of a point G such that $\triangle ABC \cong \triangle EFG$. Is there another location for G such that $\triangle ABC \cong \triangle EFG$?

20. Name the coordinates of two possible points H such that $\triangle ABC \cong \triangle FEH$.



Written Exercises

Suppose $\triangle BIG \cong \triangle CAT$. Complete.

1. $\angle G \cong$ ___?___

2. ___?___ $= m\angle A$

3. $BI =$ ___?___

4. ___?___ $\cong \overline{AT}$

5. $\triangle IGB \cong$ ___?___

6. ___?___ $\cong \triangle CTA$

7. If $\triangle DEF \cong \triangle RST$, $m\angle D = 100$, and $m\angle F = 40$, name four congruent angles.

8. Is the statement "Corresponding parts of congruent triangles are congruent" based on a definition, postulate, or theorem?

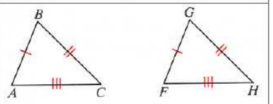
9. Suppose $\triangle LXR \cong \triangle FNE$. List six congruences that can be justified by the following reason: Corr. parts of $\cong$ $\triangle$ are $\cong$.

Section 4.2
Ways to prove Triangles Congruent

Note- instead of proving every corresponding part to two triangles congruent, we have three ways to show two triangles are congruent.

Way # 1
SSS Postulate

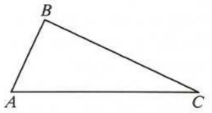
- If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.



$\triangle ABC \cong \triangle FGH$

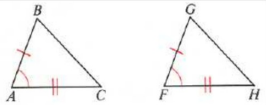
Helpful information to describe parts of triangles:

$\overline{AB}$ is *opposite* $\angle C$.
$\overline{AB}$ is *included* between $\angle A$ and $\angle B$.
$\angle A$ is *opposite* $\overline{BC}$.
$\angle A$ is *included* between $\overline{AB}$ and $\overline{AC}$.

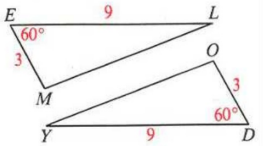


Way # 2
SAS Postulate

- If two sides and the included angle of one triangle are congruent to the two sides and the included angle of another triangle, then the triangles are congruent.

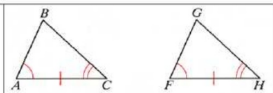


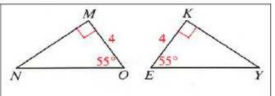
$\triangle ABC \cong \triangle FGH$ and $\triangle MEL \cong \triangle ODY$



Way # 3
ASA Postulate

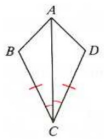
- If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.



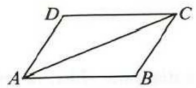


$\triangle ABC \cong \triangle FGH$ and $\triangle MON \cong \triangle KEY$

Ex 1.



16. Supply the missing reasons.
Given: $\overline{AB} \parallel \overline{DC}$; $\overline{AB} \cong \overline{DC}$
Prove: $\triangle ABC \cong \triangle CDA$



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1. ?
2. $\overline{AC} \cong \overline{AC}$	2. ?
3. $\overline{AB} \parallel \overline{DC}$	3. ?
4. $\angle BAC \cong \angle DCA$	4. ?
5. $\triangle ABC \cong \triangle CDA$	5. ?

Section 4.3
Using Congruent Triangles

We want to prove two segments or two angles in a triangle congruent.

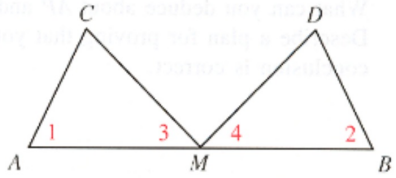
Ways to prove two segments or two angles congruent

* Identify two triangles in which two segments or angles are corresponding parts

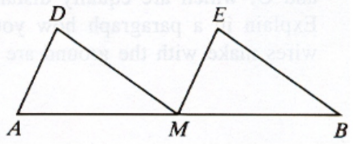
* Prove that the triangles are congruent (SAS, SSS, ASA)

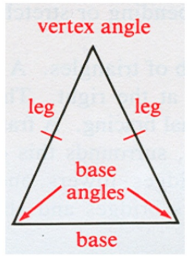
* State that the two parts are congruent, using the reason CPCTC

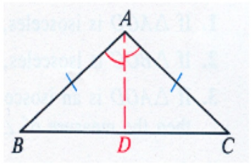
4. Given: M is the midpoint of $\overline{AB}$;
$\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$
Prove: $\overline{AC} \cong \overline{BD}$

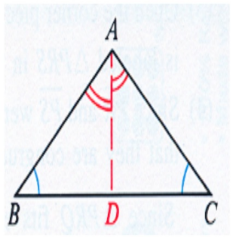


8. Given: M is the midpoint of $\overline{AB}$;
$\overline{AD} \cong \overline{ME}$; $\overline{AD} \parallel \overline{ME}$
Prove: $\overline{MD} \parallel \overline{BE}$



	Section 4.4 The Isosceles Triangle Theorems	
What is an isosceles triangle?		

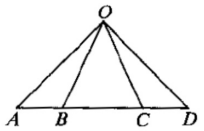
Isosceles Triangle Theorem 	If two sides of a triangle are congruent, then the angles opposite those sides are congruent.	
Corollary 1 An equilateral triangle is equianglular	Corollary 2 An equilateral triangle has three 60^0 angles	Corollary 3 The bisector of the vertex of an isosceles triangle is perpendicular to its base at the midpoint.

Theorem ■ If two angles of a triangle are congruent, then sides opposite to those angles are congruent		
Corollary An equiangular triangle is also equilateral		

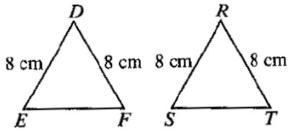
Prove the Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

1. If $\triangle AOD$ is isosceles, with $\overline{OA} \cong \overline{OD}$, then $\angle _?_ \cong \angle _?_$.
2. If $\triangle BOC$ is isosceles, with $\overline{OB} \cong \overline{OC}$, then $\angle _?_ \cong \angle _?_$.
3. If $\triangle AOD$ is an isosceles right triangle with right $\angle AOD$, then the measure of $\angle A$ is $_?_$.

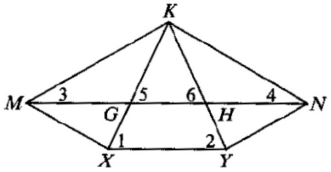


4. Given the triangles at the right, which of the following can you conclude are true?
   - **a.** $\angle D \cong \angle R$
   - **b.** $\overline{DE} \cong \overline{DF}$
   - **c.** $\overline{DF} \cong \overline{RT}$
   - **d.** $\angle E \cong \angle F$
   - **e.** $\angle E \cong \angle S$
   - **f.** $\angle S \cong \angle T$

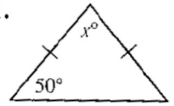


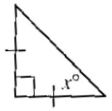
Given the two congruent angles, name two segments that must be congruent.

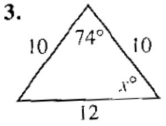
5. $\angle 1 \cong \angle 2$
6. $\angle 3 \cong \angle 4$
7. $\angle 5 \cong \angle 6$
8. Is the statement "$\overline{MK} \cong \overline{NK}$ if and only if $\angle 3 \cong \angle 4$" true or false?

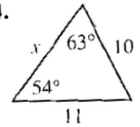


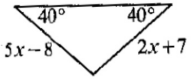
Find the value of x.

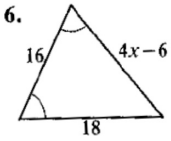
1. 

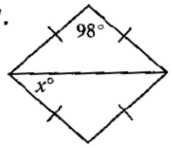
2. 

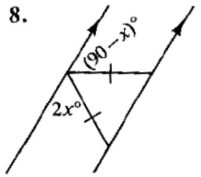
3. 

4. 

5. 

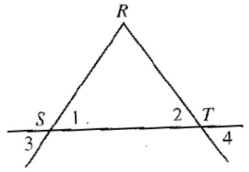
6. 

7. 

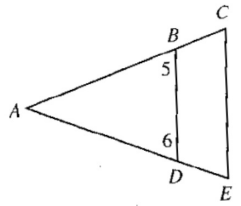
8. 

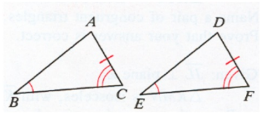
For each exercise place the statements in an appropriate order for a proof. (There may be more than one correct order.)

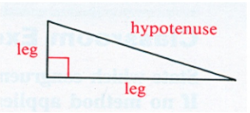
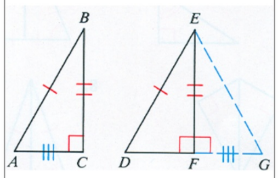
9. Given: $\overline{RS} \cong \overline{RT}$
   Prove: $\angle 3 \cong \angle 4$
   - (a) $\angle 3 \cong \angle 4$
   - (b) $\angle 3 \cong \angle 1$; $\angle 2 \cong \angle 4$
   - (c) $\overline{RS} \cong \overline{RT}$
   - (d) $\angle 1 \cong \angle 2$

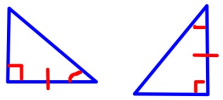
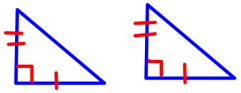
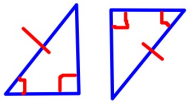


10. Given: $\overline{BD} \parallel \overline{CE}$; $\angle 5 \cong \angle 6$
    Prove: $\overline{AC} \cong \overline{AE}$
    - (a) $\overline{BD} \parallel \overline{CE}$
    - (b) $\overline{AC} \cong \overline{AE}$
    - (c) $\angle 5 \cong \angle C$; $\angle 6 \cong \angle E$
    - (d) $\angle 5 \cong \angle 6$
    - (e) $\angle C \cong \angle E$



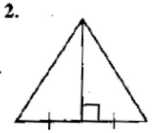
	Section 4.5 Other Methods of Proving Triangles Congruent	
AAS Theorem If 2 angles and a non-included side of a △ are congruent to cor. parts of another triangle, then the triangles are congruent		

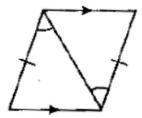
Right Triangle 		
HL Theorem if the hypotenuse and one leg of a △ are ≅ to cor. parts of another △ then triangles ≅		

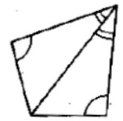
Leg-Acute Angle Method (LA)	Leg-Leg Method (LL)	Hypotenuse-Acute Method (HA)
		
(AAS, ASA)	SAS	AAS
Ways to show all triangles congruent: AAS, SAS, ASA, SSS	Ways to show right triangles congruent: HL, LA, LL, HA	

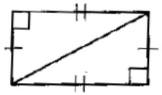
State which congruence method(s) can be used to prove the triangles congruent. If no method applies, write *none*.

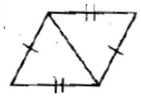
1.

2. 

3. 

4. 

5. 

6. 

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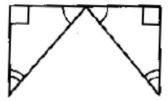
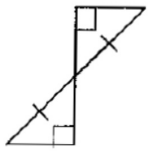
8. 

State which congruence method(s) can be used to prove the triangles congruent. If no method applies, say *none*.

1.



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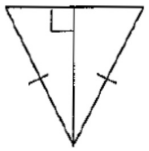

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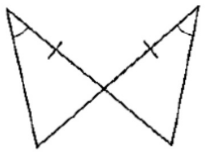

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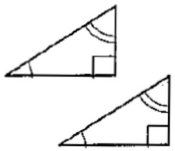

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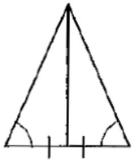

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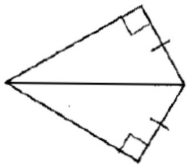

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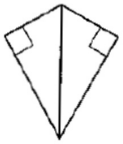

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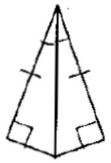

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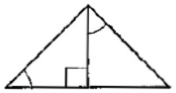
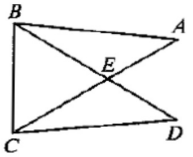
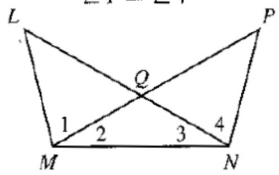


For each diagram, name a pair of overlapping triangles. Tell whether the triangles are congruent by the SSS, SAS, ASA, AAS, or HL method.

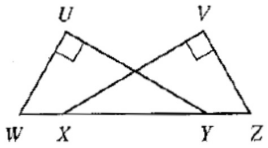
10. Given: $\overline{AB} \cong \overline{DC}$; $\overline{AC} \cong \overline{DB}$



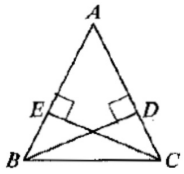
11. Given: $\angle 2 \cong \angle 3$; $\angle 1 \cong \angle 4$



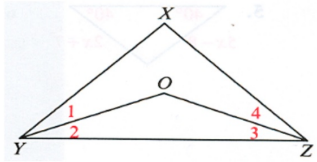
12. Given: $\overline{WU} \cong \overline{ZV}$; $WX = YZ$; $\angle U$ and $\angle V$ are rt. $\angle$s.



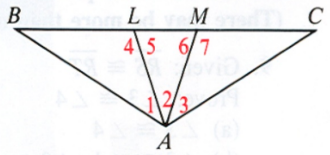
13. Given: $\angle ABC \cong \angle ACB$; $\overline{AE} \perp \overline{EC}$; $\overline{AD} \perp \overline{DB}$



18. Given: $\overline{XY} \cong \overline{XZ}$;
$\overrightarrow{YO}$ bisects $\angle XYZ$;
$\overrightarrow{ZO}$ bisects $\angle XZY$.
Prove: $\overline{YO} \cong \overline{ZO}$

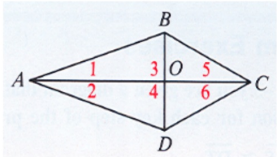


Given: $\angle 4 \cong \angle 7$; $\angle 1 \cong \angle 3$
Prove: $\triangle ABC$ is isosceles.



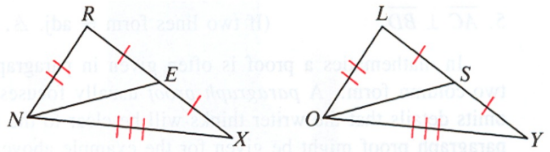
Section 4.6
Using More than One Pair of Congruent Triangles

Ex. Given: $\angle 1 \cong \angle 2$; $\angle 5 \cong \angle 6$
Prove: $\overline{AC} \perp \overline{BD}$



Ex. **you are given a diagram that is marked with given information.**

Prove: $\overline{NE} \cong \overline{OS}$



In Exercises 1–3 you are given a diagram that is marked with given information. Give the reason for each key step of the proof.

1. Prove: $\overline{AS} \cong \overline{DT}$

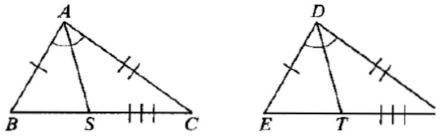
Key steps of proof:

a. $\triangle ABC \cong \triangle DEF$

b. $\angle C \cong \angle F$

c. $\triangle ACS \cong \triangle DFT$

d. $\overline{AS} \cong \overline{DT}$



2. Prove: $\overline{AX} \cong \overline{AY}$

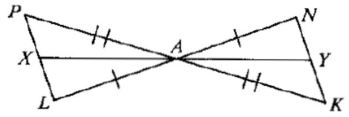
Key steps of proof:

a. $\triangle PAL \cong \triangle KAN$

b. $\angle L \cong \angle N$

c. $\triangle LAX \cong \triangle NAY$

d. $\overline{AX} \cong \overline{AY}$



3. Prove: $\angle 3 \cong \angle 4$

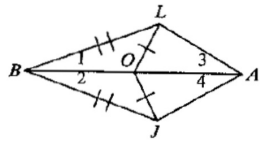
Key steps of proof:

a. $\triangle LOB \cong \triangle JOB$

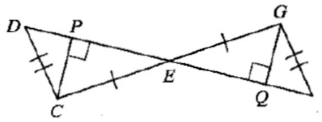
b. $\angle 1 \cong \angle 2$

c. $\triangle LBA \cong \triangle JBA$

d. $\angle 3 \cong \angle 4$

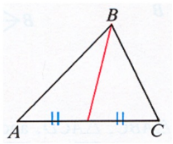


4. Suggest a plan for proving that $\angle D \cong \angle F$.

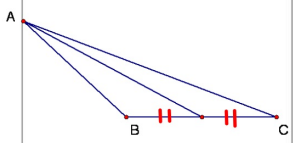


Section 4.7
Medians, Altitudes, and Perpendicular Bisectors

Median
A segment from a vertex to the midpoint of the opposite side



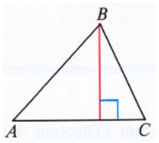
Acute

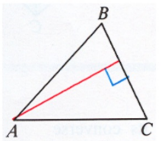


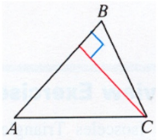
Obtuse

Altitude
The perpendicular segment from a vertex to the line that contains the opposite side

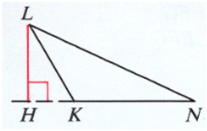
Acute

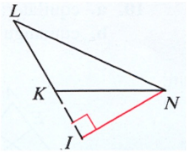


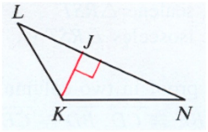




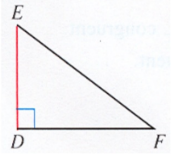
Obtuse

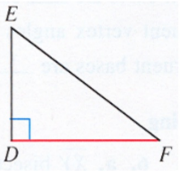


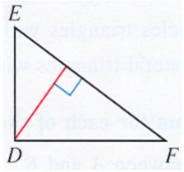




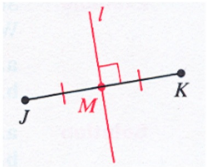
Right



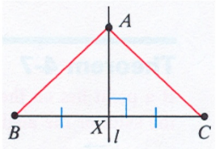




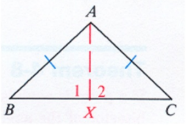
Perpendicular Bisector
A line, ray, or segment that is perpendicular to the segment at its midpoint

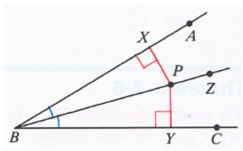
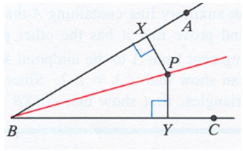


Theorem
If a point is on the perpendicular bisector of a segment, then the point is equidistant from the endpoints

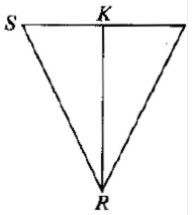


Is the converse true?

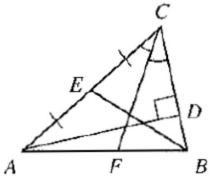


Theorem		Is the converse true?
If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle		
	Statements Reasons	

1. If K is the midpoint of $\overline{ST}$, then $\overline{RK}$ is called a(n) ___?___ of $\triangle RST$.
2. If $\overline{RK} \perp \overline{ST}$, then $\overline{RK}$ is called a(n) ___?___ of $\triangle RST$.
3. If K is the midpoint of $\overline{ST}$ and $\overline{RK} \perp \overline{ST}$, then $\overline{RK}$ is called a(n) ___?___ of $\overline{ST}$.
4. If $\overline{RK}$ is both an altitude and a median of $\triangle RST$, then:
   a. $\triangle RSK \cong \triangle RTK$ by ___?___. b. $\triangle RST$ is a(n) ___?___ triangle.
5. If R is on the perpendicular bisector of $\overline{ST}$, then R is equidistant from ___?___ and ___?___. Thus ___?___ = ___?___.

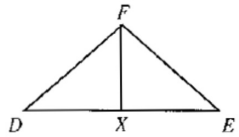


6. Refer to $\triangle ABC$ and name each of the following.
   a. a median of $\triangle ABC$
   b. an altitude of $\triangle ABC$
   c. a bisector of an angle of $\triangle ABC$

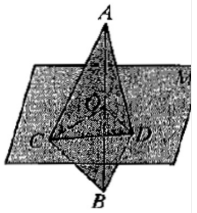


7. Draw XY. Label its midpoint Q.
   a. Select a point P equidistant from X and Y. Draw $\overline{PX}$, $\overline{PY}$, and $\overline{PQ}$.
   b. What postulate justifies the statement $\triangle PQX \cong \triangle PQY$?
   c. What reason justifies the statement $\angle PQX \cong \angle PQY$?
   d. What reason justifies the statement $\overleftrightarrow{PQ} \perp \overline{XY}$?
   e. What name for $\overleftrightarrow{PQ}$ best describes the relationship between $\overleftrightarrow{PQ}$ and $\overline{XY}$?

8. Given: $\triangle DEF$ is isosceles with $DF = EF$; $\overline{FX}$ bisects $\angle DFE$.
   a. Would the median drawn from F to $\overline{DE}$ be the same segment as $\overline{FX}$?
   b. Would the altitude drawn from F to $\overline{DE}$ be the same segment as $\overline{FX}$?



9. What kind of triangle has three angle bisectors that are also altitudes and medians?

10. Given: $\overrightarrow{NO}$ bisects $\angle N$.
    What can you conclude from each of the following additional statements?
    a. P lies on $\overrightarrow{NO}$.
    b. The distance from a point Q to each side of $\angle N$ is 13.

11. Plane M is the *perpendicular bisecting plane* of $\overline{AB}$ at O (that is, M is the plane that is perpendicular to $\overline{AB}$ at its midpoint, O). Points C and D also lie in plane M. List three pairs of congruent triangles and tell which congruence method can be used to prove each pair congruent.